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ABSTRACT

Starting with the occurrence and discussion of the importance of cavitation in turbomachines, the development of the various stages of cavitation in a machine is reviewed, followed by a discussion of the various dimensionless parameters used to describe cavitating performance in a turbomachine. The calculations of optimum inlet diameter are carried out in terms of assumed acceptable cavitation parameters. Various methods of determining the minimum cavitation number before cavitation breakdown are explored, and a free streamline model of the flow in a simplified impeller is used to obtain theoretical estimates of the extent of cavitation as a function of blade geometry and angle of attack. Some recent work of the effect of the thermodynamic properties of the fluid on the cavitation performance is examined and the various assumptions used in these theories are criticized.

NOTATIONS

$A, B, C,$	- Constants
A_1	- Inlet area (ft^2) of impeller
b	- Parameter
c	- Chord (ft)
c_m	- Meridional velocity (ft/sec)
c_p	- Specific heat at constant pressure (Btu/lb ^oR), pressure coefficient
g	- Gravitational acceleration
h	-Impeller head (ft)
h_{sv}	-Net positive suction head (ft)
Δh	- Reduction in vapor pressure (ft)
i	- $\sqrt{-1}$
k	- Cavitation number = $(p_1 - p_v)/ \frac{1}{2} \rho_1 W_1^2$
N	- Revolutions per minute
p	- Pressure
Q	- Flow rate, cubic feet per second (except in Eq. 3)
r	- Radius (ft)
s	- Entropy Btu/lb ^oR
S	- Suction specific speed (Eq. 3)
T	- Absolute temperature (^oR)
t	- Variable
u	- Velocity (ft/sec)
v	- Velocity (ft/sec)
V	- Volume (ft^3), velocity
V'	- Ratio of vapor volume to volume of mixture

w - u-iv velocity function, relative velocity to impeller

x - Coordinate, quality of vapor-liquid mixture

y - Coordinate

z - $x + iy$

α - Angle of attack, measured between inlet relative velocity and chord line

γ - Stagger angle

ζ - Complex variable

λ - Latent heat of evaporation (Btu/lb)

ν - Ratio of hub diameter to inlet diameter

ρ - Density (slugs/ft^3)

σ - Thoma's cavitation parameter (Eq. 1)

φ - Flow coefficient, meridional velocity divided by tip speed of impeller. Thus $\varphi_1 = Q/A_1 r_1 \omega$.

ψ - Head coefficient, $h/(u^2/g)$, $u = r\omega$.

ω - Angular velocity (rad./sec)

Subscripts

1 - Pertains to inlet diameter

2 - Pertains to discharge diameter

c - Characteristic velocity, perturbation quantities

f - Pertains to liquid properties

g - Pertains to vapor properties

m - Refers to meridional quantity

p - Pressure

v - Denotes vapor pressure

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I. Introduction. The occurrence of vapor and gas filled cavities within the body of a flowing liquid have been a major source of concern to the user and designer of turbomachines practically from the time these devices have come into being. This phenomenon, called "cavitation", can result whenever the static pressure of the flowing liquid falls below the vapor pressure of the liquid or in the case of a liquid saturated with dissolved gas, the bubble point. The low pressure regions of a cavitating fluid may appear to be frothy as when gas bubbles come out of solution; may consist of a series of growing and collapsing bubbles filled predominantly with vapor; or even in some cases a fairly distinct cavity attached to a solid surface may form. In all cases the hydrodynamics of the flow field can be affected to a marked degree by the presence of such cavities, and equally important, the pressure waves originated by the collapse of transient cavities in the flow may result in severe material erosion, known as cavitation damage. Various types and extent of cavitation are shown in Fig. 1 as they occur on a body of revolution.

The areas of hydrodynamic performance and damage serve as focal points for most of the cavitation research insofar as its connection with machines is concerned, that has been carried out in the United States and Europe. The literature of this subject began essentially with Rayleigh* (1) who mathematically formulated the pressure rise due to the collapse of a

* Numbers in parentheses refer to references at end of text.

spherical bubble, to explain the origin of cavitation damage. Since that time many papers have appeared that discuss physical and metallurgical aspects of cavitation damage. A comprehensive summary of these and related problems is given by Knapp [2]. The hydrodynamic performance of some simplified shapes in cavitating flow can be determined with the aid of free-streamline theory when the cavitating region is fairly distinct and well defined. The textbook example of such a flow is, of course, the plane flow past a lamina held normal to the stream, the pressure in the cavity being equal to that far upstream. This early example due to Kirchoff predicts a drag coefficient that agrees well with experiment [3]. Many more examples are found in the book by Birkhoff and Zarantonello [4].

Unfortunately, not all problems of technical importance satisfy the neat conditions of two-dimensional, steady, potential flow. As a result, most investigations of cavitation in such complicated machines as turbines, centrifugal pumps, marine propellers, etc., are perforce, experimental. The object of such researches is to establish the safe limits of operation with cavitation not only for performance but for material damage as well. As a rule a certain arbitrary reduction in efficiency such as one-half or one percent is adopted as adequate insurance against cavitation damage. The acceptable deterioration in performance is, however, dependent upon the length of anticipated service (and thus extent of damage) and upon the particular application. Thus, for example, the extensive cavitation permitted in a modern missile propellant pump would not be tolerated in the pumping plant of a major irrigation project. It is always necessary, however, to be able to establish safe hydrodynamic limits of operation with cavitation, and for this reason interest in the present paper will be centered on the hydrodynamic aspects of cavitation rather than on material damage.

In the following sections several aspects of the development of cavitation in a turbomachine will be taken up. Some of the current theories of design for cavitating flow will be discussed and finally, some aspects of the important problem of similarity in a cavitating flow will be brought out.

II. The Development of Cavitation in a Machine. The lack of detailed flow observations in machines operating with various degrees of cavitation has hampered real understanding of this problem. Visual observations of cavitating flow in a centrifugal pump, for example, are practically non-existent in the literature. Most efforts in this subject have been concerned with the interpretation of overall performance measurements. One of these is due to Gongwer [5]. The situation is scarcely better with axial flow pumps. The early paper of Tenot [6] showing photographs of cavitation development in an axial flow pump is interesting. A criticism of Tenot's work, together with additional photographs of cavitation development in an axial pump, is given in a less available report [7]. A more detailed and comprehensive photographic survey of cavitation phenomena in an axial inducer pump (a pump characterized by extreme solidity to achieve great resistance to cavitation failure at the possible expense of efficiency) is given in [8] again, unfortunately, not a part of the standard literature. The sequence of events as found in these works bears several features in common and it is anticipated that the general features of cavitation in a radial machine will probably resemble that in an axial machine. However, absence of sufficient photographs of cavitation in a centrifugal impeller precludes any definite statement on this matter.

These problems are also reviewed in a recent paper by Winternitz [9] who discusses in considerable detail the inception of cavitation, its subsequent development, typical values of cavitation parameters and some aspects of the damage problem. An extensive bibliography is also provided. For an axial flow pump at a constant flow rate and rotative speed it is found that as the inlet pressure is reduced, the cavitation progresses through several recognizable stages enumerated below [7], [8]:

(a) Inception: The first appearance of cavitation is always in or near the region of lowest pressure. If the flow is separated or if there is a strong tip clearance flow, cavitation may commence in the stream usually appearing in a tip vortex which may or may not be attached to the blade. Figure 2 shows such a case. When the tip vortex does not form, cavitation then appears first on the blade surface at or near the position of minimum pressure. Since inception plays an unimportant role in the hydrodynamic performance of a turbomachine, it will not be further elaborated upon herein and for a discussion of the problem of cavitation scaling reference 10 should be consulted.

(b) Partial Cavitation: With continued reduction in inlet pressure the cavity grows. The tip vortex region, if it occurs, becomes attached to the blade and the region of surface cavitation gradually extends over the chord of the blade. Generally speaking, when the length of the cavity is less than about one-half chord, no gross effect on head generation is observed provided the cavitation commences from the leading edge of the blade. Still, however, the head may be lowered by several percent and the efficiency perhaps even more. If the impeller solidity is large (say 1.5) a condition of non-steady flow may also develop, depending upon the particular blade design. Also, observation of partial cavitation on Karman-Trefftz airfoils when the cavity is about one-half chord long [11] indicate that pumps of low solidity will probably be subject to similar non-steady cavitation flows. The exact origin of the unsteadiness is as yet unknown. It is, however, a phenomenon of some importance as pumps that operate with such flows are subject to considerable vibration.

(c) Impending Failure or Cavitation Breakdown: With further decrease in inlet pressure, the length of the cavitating region (steady or unsteady) rapidly approaches the chord length and the head of the unit may suddenly drop to a very low value. For practical purposes the pump may be said to have failed. Examples of such behavior are common in the literature and the representative plots in Ref. 12 may be cited as being typical. It is just this condition that the designer must be able to estimate in order to determine the degree of safety of a particular application. Moreover, there are circumstances such as in the regulation of condensate pumps in which it is desirable to operate with various heads within the region of "cutoff".

The foregoing development of cavitation as it occurs in an inducer impeller is illustrated in Fig. 3 taken from Ref. (8). It is not clear from these photographs but most of the visible cavitation occurs in a rather thin annulus near the impeller tip. The gradual development of the cavity is in this case regular and no unsteady flow occurs. It will be noticed, however, that extremely low cavitation numbers (see notation) are achieved before breakdown. It will be seen later that this is a consequence of the low blade angle and high solidity of this configuration. The dimensionless head coefficients are plotted vs the cavitation numbers for various flow rates (Fig. 4) for this impeller. It is clear from this diagram that to a certain extent, the acceptable safe limit is arbitrary, provided that it exceeds the breakdown limit. It becomes therefore imperative to have some measure of the breakdown limit as well as an understanding of the physical processes implicit in the use of the dimensionless parameters such as the cavitation number used in this diagram.

An interesting series of photographs of cavitation in a Kaplan turbine [13] show many of the same features discussed above for an inducer pump impeller.

III. Estimates of Cavitation Performance

1. Similarity Parameters for Cavitation: The cavitating performance of a hydraulic machine in the field is usually expressed in terms of suction lift at a given flow rate and speed. However such a presentation is not useful for application to other pump sizes, speeds or lifts, and for that reason representation in terms of dimensionless similarity parameters is always preferred. Various parameters are now in vogue with hydraulic engineers for this purpose. A convenient one (and one of the first) is Thoma's σ where

$$\sigma = hsv/h. \tag{1}$$

In this equation hsv stands for the net positive suction head and this is equal to the inlet total head (absolute not gauge) minus the vapor pressure of the fluid. h is the non-cavitating head of the pump. At a given flow rate coefficient (see notation) it can be shown that constancy of σ implies constancy of the cavitation condition provided other physical effects do not occur (such as important Reynolds number effects, non equilibrium thermodynamic and surface tension effects, etc.). However, since the magnitude of the generated head affects only to a slight degree the distribution of pressure in an impeller of moderate solidity, the magnitude of σ is not a sensitive indicator of cavitation performance and varies greatly with impeller type. For this reason the cavitation number

$$k = (p_1 - p_v) / \tfrac{1}{2}\rho\, w_1^{\,2} \tag{2}$$

or the suction specific speed

$$S = N\sqrt{Q} / (hsv)^{3/4} \tag{3}$$

are better parameters. In Eq. (2), p_1 is the inlet static pressure to the impeller, p_v the vapor pressure, ρ the density and w_1 is the inlet relative velocity at the tip of the impeller. The algebraic relation between (1), (2) and (3) is brought out more directly by introduction of the dimensionless head coefficient ψ and flow coefficient φ (see notation). Making the indicated substitutions it is found that

$$k = \frac{2\psi_2\sigma\left(\frac{r_2}{r_1}\right)^2 - \varphi_1^{\,2}}{1+\varphi_1^{\,2}} \, . \tag{4}$$

For axial flow machines $r_2 = r_1$; thus, this ratio disappears. The suction specific speed is

$$S = \frac{8150\, \varphi_1^{1/2}}{\left\{(1+\varphi_1^2)\, k + \varphi_1^2\right\}^{3/4}} \sqrt{A_1/\pi r_1^2} \qquad (5)$$

a form suitable for either centrifugal or axial pumps. For the latter, the last square root member becomes $\sqrt{1 - \nu^2}$. In Eq. (3), N is to be measured in RPM, and Q in gallons per minute (U. S.). S, as found is not dimensionless but would be so with the introduction of $g^{3/4}$ into the denominator ($g = 32.2$ ft/sec^2, the standard gravitational constant).

From these relations it is seen that if the operating point of a given turbomachine is fixed (φ, ψ const.) constancy of σ implies constancy of k or S.

2. Estimates of Cavitation Parameters: The problem now is simply to be able to estimate values of k or S such as to ensure freedom from severe performance deterioration due to cavitation. This is, understandably, a most difficult problem. What can be done for simplified shapes (for example, a cascade of hydrofoils) is to estimate the inception of cavitation by equating the minimum pressure on the blade to the vapor pressure of the fluid. It is particularly simple to do this for pump blades of low solidity and for this case Bowerman [14] has devised a neat procedure for designing pumps of given duty (head, flow, speed) for given diameters with optimum resistance to (the inception of) cavitation.

However, if it is known that a certain type of pump will operate successfully with a particular value of k, independent of φ or ψ, a special case of Bowerman's theory results which enables one to determine easily for example the optimum diameter of the impeller eye. Such considerations were first put forward by Brumfield [15] Pfleiderer [16] and Ross [17]. In fact due to its simplicity this calculation (by Ross) is worth repeating:

Let the inlet total pressure be p_o. Then in the notation of Fig. 5,

$$p_1 = p_o - \frac{\rho}{2} c_m^2$$

and with (2)

$$p_1 = \frac{\rho}{2} (V_1^2 + c_m^2) k - p_v$$

or

$$p_o = \left[\frac{\rho}{2} \quad kV_1^2 + c_m^2 (1 + k) \right] - p_v$$

The optimum diameter of the impeller eye for a given speed and flow rate will occur when p_o is a minimum and the minimum blade pressure equals the vapor pressure p_v.

$$0 = \frac{d}{dr} \left[k\, r^2 \omega^2 + (1 + k) \frac{Q^2}{\pi^2 r^4} \right]$$

is therefore the condition to be satisfied for the optimum radius. Carrying out the indicated operations results in relation

$$r_{1_{opt}} = \left[\frac{2 (1 + k) Q^2}{k \pi^2 \omega^2} \right]^{1/6} \qquad (6)$$

A more interesting result is obtained when the optimum flow rate coefficient is found

$$\varphi_{1_{opt}} = \sqrt{\frac{k}{2 (1 + k)}} \qquad (7)$$

and with this

$$S_{opt} = \frac{5060}{k^{1/2}(1+k)^{1/4}} \sqrt{A_1/\pi r_1^2} \qquad (8)$$

Although (8) is for the optimum eye diameter, it will be found that (6) also maximizes (5) for the given conditions. Thus, the radius given by (6) is the best radius, the flow coefficient of (7) and the suction specific speed of (8) are the best that can be achieved with the given value of k. It is interesting to determine the order of magnitude of these quantities for some realistic estimates of k. It can be seen upon reference to Fig. 4 that k = 0.03 is certainly achievable. For this value of k the optimum flow coefficient is from Eq. (7), $\varphi_{1_{opt}} = 0.12$ and the value of S_{opt} is 26,800. (It is assumed that $\nu = 0.4$.) This value is quite high and would only be used in machines of short life.

These numbers are in interesting contrast to more conventional pumps. For example, the data of (18) indicate that a value of k of about 0.31 can be expected near breakdown for a well-designed centrifugal pump. Under this condition, the best suction specific speed with $A_1/\pi r_1^2 = 1$ that could be expected would only amount to about 8400. This value is close to the Hydraulic Institute standard for single suction centrifugal pumps [(19)].

The development sketched above can be modified easily to include upstream friction loss, flow distortion and pre-rotation of the flow. The advantages of pre-whirl in achieving high suction specific speeds (and therefore good cavitation resistance) is clearly brought out in a recent paper by Wislicenus [(20)]. However, in all of these cases <u>a priori</u> knowledge of the acceptable cavitation number is required. As mentioned earlier, this can be obtained analytically for inception of cavitation only in the case of some simplified flows such as a cascade. Even then considerable labor is required, for example, see (21). The growth of the cavitating region in an impeller as the cavitation number is lowered and its correlation with performance is therefore a problem of great importance as this information would provide some basis for the calculations above. It should come as no surprise that the theoretical difficulties of treating the cavitating flow in a general turbomachine are practically insurmountable. For some simplified shapes, e. g., the impeller shown in Fig. 3, some progress has been made recently and this will be reviewed in the next few paragraphs.

As the simplest meaningful example that can be treated, the plane potential flow through a cascade of flat plates will be discussed in detail. The cavity is assumed to be attached to the leading edge of the blade (Fig. 6) and for reasons of simplicity, the length of the plates is taken to be infinite. The flow is frictionless, and the cavity is bounded by a free streamline on which the fluid velocity is constant. The position of the streamline and length of the cavity are unknown before hand. The problem to be solved is to determine the extent of the cavity (compared to the blade spacing 2π) as a function of the cavitation number k, the angle of attack α and the stagger angle γ. The solution can be obtained by hodograph methods which are abundantly treated in Ref. 4. However, because of the recent interest in the application of linearized free streamline theory to problems that are handled only with considerable difficulty by exact methods, the linearized theory will be used herein.

Briefly, the method assumes that the angle of attack and the slope of the streamlines are small compared to unity (as in thin airfoil theory).

As a consequence, it may be assumed that the basic flow through the cascade is only slightly perturbed from its value far upstream. Products and squares of the perturbation velocity components are then neglected, so that the pressure becomes a linear function of the perturbation velocity in the stream direction. Boundary conditions are then applied on the chord and not at the actual position of the streamline. The method is approximate but because of its simplicity, it is of great utility. The linearized theory as applied to thin airfoils is covered in Ref. (22), and Parkin [23] treats in detail a great many free streamline problems by this method.

3. Discussion of Theoretical Results: The details of the problem outlined above may be found in Appendix I, and for the purposes of the present discussion the salient results will be repeated below. First, however, it is of interest to consider the results of two limiting cases; namely, the limit of zero cavity length to spacing ratio and infinite cavity length. It may be shown from Eqs. A-6 and A-11 that for small cavity lengths

$$k \longrightarrow \frac{4 \sin 2a}{\sqrt{2\pi}} \sqrt{\cos \gamma / (c/2\pi)} .$$

Thus, as the cavity becomes very short, k approaches infinity in accordance with the known limit for fully wetted flow. On the other hand, as the cavity length approaches infinity, Eq. (A-11) shows that k becomes asymptotic to the limiting value

$$k = 2 \sin a \cos (\gamma + a) / (1 + \sin \gamma). \tag{9}$$

This result can also be obtained from momentum considerations when it is recalled that the flow is frictionless and that there must be no force parallel to the plane. Equation (9) is useful since it gives the least possible value of the cavitation number for flow through a cascade. It is seen that $k = 0$ for $\alpha = 0$ (it being assumed that there is no thickness to the blades), but in addition k is also zero for $\gamma + \alpha = \pi/2$. This finding, perhaps surprising at first, occurs only at zero flow through the cascade and does not therefore represent a flow of interest. The maximum of k in Eq. (9) is perhaps a more realistic condition and this occurs when $\alpha = (\pi/2 - \gamma)/2$ and has the value $k_{max} = (1 - \sin\gamma)/(1 + \sin\gamma)$. This simple result clearly shows that large stagger angles are more conducive to cavitation resistance, and this observation is generally borne out in practice. In addition, there is some suggestion (5), (8) that the value of k_{max} just quoted is a fair measure of the minimum cavitation number achievable in an inducer pump or inlet portion of a centrifugal pump. Specifically, the test results of (8) show that twice the value of k so determined is a conservative estimate of the breakdown cavitation number in inducer pump devices.

The dependence of cavity length upon cavitation number for various values of γ and for an angle of attack of six degrees is shown in Fig. 8. The dominant effect of stagger angle is clearly brought out in the diagram. Equally interesting is the fact that for all practical purposes the asymptote of minimum cavitation number is reached when the cavity is one and a half blade spacings. The "knee" of the curve occurs at about one blade spacing for most stagger angles. This finding indicates that excessive solidity is not required to achieve high suction specific speeds and partial corroboration of this conclusion was observed in Ref. (8). Direct experimental evidence confirming the general behavior shown in Fig. (8) is, however, lacking. To investigate this point visual observations of the cavitating flow in an inducer with a nine degree tip angle at an angle of attack for which three dimensional flows were not large, were made and compared to the theory. These results are shown in Fig. 9. The agreement, though not too good, certainly bears out the general features of the theory. It would therefore be of great interest to extend observations of this type to centrifugal pumps and to additional blade angles.

The simple analysis indicated above can be extended fairly easily to account for finite length of the blades. An example of such a calculation for cavity lengths greater than the chord length is given in Ref. (24). Also, curved blades can be treated as well [25]. However, the effects of rotational flow on the development of cavitation have not yet been determined (e. g., that which occurs on the meridional streamline surfaces in a centrifugal impeller). Much interesting and valuable work remains to be done in this area.

IV. Cavitation Similarity. In the foregoing sections it was tacitly assumed that the pressure in the cavity was equal to the vapor pressure of the bulk liquid. Furthermore, it was also assumed that cavitation occurs as a pocket or bubble attached to the blade surface. These assumptions now have to be examined in more detail, especially as recent experimental findings on pumps operating with hot water, some hydrocarbons and cryogenic fluids such as liquid nitrogen, show pronounced differences in their cavitating performance. As an illustration, Fig. (10), taken from Stahl's paper [26] compares the performance of the same pump operating with cold water and water at 294°C (the flow rate and speed are kept constant). It can be seen that the cavitating performance with hot water is appreciable better. In fact, the results of numerous investigations [27], [28], [29], [30] show that all pure liquids containing no dissolved gas exhibit a cavitating performance equal to or better than that of cold water. One possibility that immediately suggests itself is that the pressure p_v assumed to occur in the cavity, is lower than the vapor pressure of the bulk fluid, resulting in a higher cavitation number and therefore a less severe condition to be met by the pump. Various rough physical arguments can be put forward to support the view that this actually happens. For this purpose, it is necessary to have some idea of the processes involved in cavitation. One such model (previously discussed) is that of the stable cavity attached to the blade. It would seem to be reasonable that some of the vapor within such a cavity is entrained by the violent turbulent mixing processes occurring at the end of the cavity. In order to maintain the cavity, liquid must then be evaporated into it from the surface of the flow. The process of evaporation cools the inner bounding surface of the cavity to the degree necessary to cause the heat required for evaporation to flow to the surface. Thus, according to whether the density of the vapor (and hence mass of evaporated liquid), latent heat of evaporation, and entrainment rate are large or not, the cooling effect will be large or perhaps even negligible. Other models of the cavitation process also lead to the same conclusion. Thus, for example, it might be argued that cavitation occurs in the flow as the nuclei that are present in the liquid move with the fluid into the low pressure regions and grow there. The rate of growth of such a bubble depends upon the extent to which it is superheated in the low pressure region since it is always necessary in a pure liquid to have a certain degree of superheat for a finite growth rate. The actual process as it occurs in a machine probably lies between these extremes. In fact, as Salemann points out, there is indirect evidence to show that the first model occurs when pumping fluids such as cold water and the second for fluids such as hot water that show the pronounced "thermal" effect.

Considerable effort has been devoted by the groups listed in references (26) to (30) (as well as others not listed) to establish the correct similarity laws for cavitating flows with various liquids. Most of the workers take the point of view that in the cavitation process, a certain amount of vapor is formed within the inlet portions of the impeller and that for similar cavitating performance, the ratio of the volume of vapor so formed to the volume of the vapor-liquid mixture must be the same. It is also assumed that the vapor and the bulk liquid are in thermal equilibrium. The evaporation of the liquid is accompanied by a decrease in bulk temperature, and a corresponding reduction of the vapor pressure. The reduction in vapor pressure for a given ratio of vapor to liquid volume will depend upon the particular fluid and accordingly the fluid will be said to have a low or high tendency to cavitate as the depression is high or low respectively. Or conversely, the vapor to liquid volume ratio for a given decrease in vapor pressure may be said to indicate the susceptibility of a fluid to cavitate [(27)].

The analyses of the depression of the vapor pressure (or reduction in the net positive suction head required for similar cavitation) presented in references (27), (28), (29) although differing in some details, are all essentially the same; and it can be shown, for example, that the considerations of Jacobs [(28)] are fully equivalent to the following problem of "thermostatics". An insulated beaker with a tightly fitted piston is completely filled with a unit mass of saturated liquid. The piston is then slowly withdrawn until a certain given volume of the vapor is formed. The pressure of the resulting mixture is then to be determined.

This problem as stated above is equivalent to calculating the expansion of the vapor from the point of view of an observer moving with the fluid, provided that there is no friction or heat transfer. The thermodynamic equations for the above problem will now be solved using the symbols of Jacobs.

Let the quality (or mass friction of the vapor) of the mixture be x, and subscripts (1) and (2) denote the beginning and end of the process respectively. Since the process is adiabatic and frictionless, it is reversible and therefore isentropic. Thus

$$s_2 = s_{f2} + x\, s_{fg2} = s_{f1} \tag{10}$$

with the usual thermodynamic notation. It is assumed that the liquid has a negligible coefficient of thermal expansion, so that the entropy change of the liquid is $s_{f1} - s_{f2} = c_p \, \ln (T_1/T_2)$. Writing $s_{fg} = \lambda/T_2$ one has

$$\lambda \frac{x}{T_2} = c_p \, \ln (T_1/T_2) . \tag{11}$$

To obtain the connection between the temperature change and the pressure, the Clausius-Clapeyron equation must be used, i. e.,

$$\frac{dp}{dT} = \frac{\lambda}{v_{fg} \, T} . \tag{12}$$

If the temperature change $\Delta T = T_2 - T_1$ is small compared to T_2, the logarithm of Eq. (11) can be expanded, and only the leading term retained. When the temperature increment is substituted from (12) the expression

$$\Delta p = \frac{\lambda^2 x}{v_{fg} \, T c_p} \tag{13}$$

is obtained, in agreement with Jacob. The depression Δp of the vapor pressure necessary to create the vapor is usually expressed in feet of the liquid, i. e., $\Delta p = -\Delta h / v_f$. Furthermore, the quality x can be eliminated in favor of the ratio of volume of vapor to volume of liquid

$$B = \frac{\text{volume vapor formed}}{\text{volume of liquid}} = \frac{x v_{fg}}{v_f} = \frac{V}{v_f}, \tag{14}$$

or the ratio of volume of vapor formed to the volume of the mixture

$$V' = \frac{\text{volume of vapor formed}}{\text{volume of mixture}} = \frac{x\, v_{fg}}{v_f + x v_{fg}} \tag{15}$$

resulting in

$$\Delta h = \frac{\lambda^2 v_f^2}{v_g^2 T c_p} \cdot \frac{V'}{1 - V'}, \tag{16}$$

and

$$B = \Delta h / \frac{\lambda^2 v_f^2}{v_g^2 T c_p}. \tag{17}$$

In these latter expressions the approximation $v_{fg} = v_g$ is also used.

If it is now assumed, as Jacobs does, that similar cavitating conditions in a given pump result when V' is the same, then whatever the fluid, the ratio $\Delta h / (\lambda^2 v_f^2 / v_g^2 T c_p)$ from Eq. (10) should be constant for similar flows to occur. Thus the depression of the vapor pressure for any fluid, Δh, necessary to cause the requisite amount of vapor to form, and by supposition, to obtain the same hydraulic performance of the pump, can be found if the value of Δh is known for any other fluid and if the thermal properties λ, c_p, v_f, v_g are known for each of the fluids.

On this basis Jacobs is able to correlate fairly well the cavitating performance of a machine pumping liquid hydrogen and liquid nitrogen.

Alternatively, Stepanoff chooses the value of B for $\Delta h = 1$ ft as an arbitrary measure of the susceptibility of the fluid to cavitate. Since this quantity shows wide variations from one fluid to another, it is of interest to tabulate several values.

According to the information of Table I, propane and hydrogen should show little deterioration in performance due to cavitation since the volume of the vapor formed is small. Nitrogen and hot water follow in that order. However, the excessively large value of B for cold water indicates that essentially no depression could actually occur in the pump. Tests conducted with cold water, therefore, give "conservative" results.

Table I. Cavitation Susceptibility Parameter B for Various Fluids

Substance	Temperature	B for $\Delta h = 1$ ft.
Water	70^{o}	1800.
Water	100	353.
Water	180	7.87
Water	212	2.08
Water	250	0.66
Water	300	0.16
Butane	70	0.17
Butane	31	0.57
Propane	100	0.008
Propane	$44^{o}F$	0.92
Hydrogen	$36^{o}R$	0.01
Nitrogen	$138^{o}R$	0.40
Oxygen	$162^{o}R$	0.69
Freon 11	$75^{o}F$	1.63
Freon 22	$91^{o}F$	1.28
Freon 113	$118^{o}F$	1.82

Despite the relative success of Jacobs in correlating the performance of nitrogen and hydrogen and that of Stepanoff in his work, it is doubtful that these simple static considerations adequately represent the cavitating process.

The constancy of the vapor to mixture volume ratio, V', as a criterion of cavitating similarity was challenged by Salemann on the basis of his measurements using a given pump operated with butane, Freon 11, and water. In fact, he found that if Δh was determined from the experiments and V' calculated from the thermodynamic relations outlined above, that V' was a function principally of the vapor pressure of the fluid varying by a factor of about four when the vapor pressure of the flowing liquid is varied by about a factor of 20. It is as a result of this finding that Salemann proposes that cavitation in cold water occurs largely as an attached cavity, whereas the large depression Δh necessitated by the higher vapor pressure fluids results in an extensive region of superheated liquid within the impeller in which a more or less homogeneous distribution of separate bubbles occurs. Intermediate states contain bubbles within the fluid as well as an attached cavity. Thus, the ratio V' need not apply to the entire flow field but only to the liquid adjacent to the cavitation and the restriction of constant V' as a condition of cavitation similarity can then be dropped.

This ingenious suggestion may be correct. It is also entirely possible that the "static" model of flow sub-cooling outlined above does in fact adequately represent the experimental cases reported. It is significant, however, that neither the rotative speed nor pump size were varied in any of these tests. Moreover, the pumps were fairly small, all 3550 rpm, the maximum flow rate not exceeding 400 gallons per minute. Without a more firm understanding of the mechanics of the cavitating process, one would be hesitant to extrapolate these results to a machine, for example, of ten times the flow rate or ten times the speed. It is difficult to escape the conviction that the time and size scales play an important role in the development of cavitation, for the average time spent by a particle in the low pressure regions of a typical centrifugal impeller is only a few milliseconds. With moderate superheats, this time is of the same order as that needed to grow a nucleus in water to an appreciable size [30]. As Plesset and Zwick point out in this series of papers, the rate at which a superheated vapor bubble grows depends upon the vapor density, latent heat, specific heat of the liquid (all quantities found in Eq. 16) and in addition, the thermal conductivity of the fluid. The volume of vapor formed per unit mass of liquid will depend upon the individual growth rate of an isolated bubble and in addition upon the number of nuclei present in the fluid. The concentration of nuclei is thought to be strongly dependent upon temperature and may well vary widely from one fluid to another. The history of the fluid may also affect the concentration and, consequently, the cavitating performance [32].

It is quite conceivable that the dynamics of individual bubble growth and distribution of nuclei might in some cases result in the volume of vapor calculated above for the static problem. In any case, it seems reasonably clear that the problem is much more complex than the simple result of Eq. (13) would indicate, and that to decide between the various possibilities outlined above more experimental work must be undertaken. However, because of the complexity of the phenomena to be investigated, the experimental apparatus should be considerably more simple than that of a centrifugal pump. In addition, it is imperative that the flow be visually observed when cavitating to establish what processes do occur for various fluids, at various temperature, fluid velocities and cavitation numbers.

Interest in the points just raised above led to the initiation of a modest research program at the Hydrodynamics Laboratory at the California Institute of Technology. A small, recirculating test circuit was constructed that could be filled with water or fluids such as Freon 113. A transparent working section of glass about one inch in diameter and four inches long is provided to view the flow. Small objects such as plates or discs, are mounted in the working section to form a cavity. The temperature, pressure and velocity can then be varied to see what conditions are necessary to maintain similar geometry of the flow--if indeed that is possible. Of more direct interest is the pressure within the cavity. This is measured by a "U" tube manometer as the difference between the vapor pressure of the bulk fluid, provided by a small closed vessel containing some of the working medium, and the pressure within the cavity itself when the latter is sufficiently clear of froth to permit such a measurement to be made. The vapor pressure bomb and manometer are all enclosed and surrounded by the hot flowing medium in a large stilling chamber upstream of the working section. Unfortunately, experimental results obtained from the apparatus just described are not yet available. However, it is hoped that some information bearing on this problem will soon be forthcoming, and further, that it may contribute to another successful future symposium.

Acknowledgment

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Appendix I. Partial Cavitation in a Cascade of Semi-Infinite Flat Plates

<u>I. Theory.</u> Figure 6 shows the cascade geometry. The boundary conditions on the cavity are placed on the upper portion of the chord, which is regarded as a slit in the physical plane. The two halves of the slit are shown slightly separated for clarity. The flow approaches the cascade with magnitude V_1 inclined at the angle α to the chord, and ultimately becomes parallel to the chord. It is convenient to define a pressure coefficient based upon the cavity pressure p_c. Thus

$$c_p = (p - p_c)/\frac{\rho}{2} V_1^{\,2} = (V_c^{\,2} - V_1^{\,2})/V_1^{\,2} \quad . \tag{A-1}$$

On the cavity $p = p_c$ so that $c_p = 0$ there. However, from the definition of the cavitation number and the Bernoulli equation

$$(p_1 - p_c) \,/\, \frac{\rho}{2} V_1^{\,2} = k = (V_c^{\,2}/V_1^{\,2}) - 1 \quad .$$

V_c is regarded as the characteristic velocity in the vicinity of the cascade blades. Small perturbations with respect to V_c in the horizontal and vertical directions are symbolically written as

$$V = (u, v) = (V_c + u_c, \; v_c)$$

where u_c, v_c are much smaller than V_c. At a general point in the flow then

$$c_p = \left[V_c^{\,2} - (V_c^{\,2} + 2V_c u_c + u_c^{\,2} + v_c^{\,2}) \right] \frac{1}{V_1^{\,2}} \approx - \frac{2V_c u_c}{V_1^{\,2}} \; , \tag{A-2}$$

The free streamline condition is therefore approximately satisfied by setting u_c equal to zero.

<u>Boundary Conditions:</u> The boundary conditions on the velocity function $w = u - iv$ are as follows:

(a) $v = v_c = 0$ on the wetted portion of the blade (i.e., no flow through the surface).

(b) $u_c = 0$ on the cavity or $u = V_c$.

(c) $w \longrightarrow V_1 e^{-i\alpha}$ far upstream (A-3)

(d) It is assumed that the cavity is closed.* This condition is equivalent to the statement that there is no net source strength in the flow.

It can be shown that conditions (a) to (d) lead to a unique solution. Since w is an analytic function of $z = x + iy$, the methods of conformal mapping can be used to obtain the solution to the boundary value problem given by Eqs. (A-3).

Mappings:

The array of slits in the z plane is transformed onto the real axis and upper half of the unit circle in the ζ plane (Fig. 7) in such a way that the wetted portions of the blade lie along the real ζ axis exterior to the circle. The portion of the slit corresponding to the cavity is the upper half of the unit circle in this plane. The region exterior to the unit circle and in the upper half ζ plane is transformed into the entire z plane. The sequence of mappings used is

---

* Models of free streamline flow that do not involve closed cavities are discussed by Parkin in (21).

$$z = e^{i\gamma} \ell n \left\{ 1 - te^{i\left(\frac{\pi}{2} - \gamma\right)} \right\} + e^{-i\gamma} \ell n \left\{ 1 - te^{-i\left(\frac{\pi}{2} - \gamma\right)} \right\} \tag{A-4}$$

and

$$\left(t - \frac{b}{2}\right) - \frac{1}{b} = \frac{1}{4}\left(\zeta + \frac{1}{\zeta}\right). \tag{A-5}$$

All of the slits in the z plane are transformed into the real axis of the t plane. A certain length (b) of this axis comprises the cavity portion. Equation (A-5) will be recognized as a Joukowsky transformation which takes this segment into the upper half of the unit circle. The point $t = e^{i\left(\frac{\pi}{2} - \gamma\right)}$ corresponds to negative infinity in the z plane and to ζ_1 in that plane. The connection between the cavity to spacing ratio $c/2\pi$ and the parameter b is got by putting $t = b$ in (A-4) to obtain

$$\frac{c}{2\pi} = \frac{1}{2\pi}\left\{ \cos\gamma \, \ell n \left(1 + b^2 - 2b \sin\gamma\right) + 2\theta_1 \sin\gamma \right\} \tag{A-6}$$

where

$$\theta_1 = \tan^{-1}\left[b \cos\gamma / (1 - b \sin\gamma) \right] .$$

Velocity Function:

General procedures are available to solve mixed boundary of the type described by Eq. (A-3) and some of them are mentioned in (23). Such an approach is not needed here, for it will be seen that the function

$$w = u - iv = \frac{A}{\zeta + 1} + \frac{B}{\zeta - 1} + C \tag{A-7}$$

satisfies all of the requirements of the present problem when constants A, B, C are suitably evaluated. The singular terms correspond to sources (sinks) at the leading edge of the hydrofoil and at the termination of the cavity. In addition (A-7) satisfies the here-to-fore unmentioned physical requirement on the flow, namely, that the minimum pressure in the flow be on the cavity.

Boundary condition (3a) is clearly satisfied by (A-7) if A, B, C are real since real ζ corresponds to the wetted portion of the blade. Condition (3b) is fulfilled if

$$V_c = \frac{1}{2}(A-B) + C = V_1 \sqrt{1-k}$$

since on the unit circle $\zeta = e^{i\theta}$ and

$$w(\zeta = e^{i\theta}) = \frac{A}{2}\left(1 - i \tan\frac{\theta}{2}\right) - \frac{B}{2}\left(1 + i \cot\frac{\theta}{2}\right) + C .$$

Condition (3c):

$$V_1 e^{-i\alpha} = \frac{A}{\zeta_1 + 1} + \frac{B}{\zeta_1 - 1} + C$$

where ζ_1 is the point corresponding to $t = e^{i(\frac{\pi}{2} - \delta)}$. To evaluate this expression it is convenient to transfer attention to the t plane by means of finding ζ in terms of t from (A-5). After this is done, the complex velocity function in the t plane is found to be

$$w(t) = \frac{A-B}{2} + C + \frac{B}{2}\sqrt{\frac{t}{t-b}} - \frac{A}{2}\sqrt{\frac{t-b}{t}} . \qquad \text{(A-8)}$$

Application of (3c) then leads to a complex expression which when separated into real and imaginary parts gives

$$V_1 \cos \alpha_1 = \frac{A - B}{2} + C + \frac{\cos(\theta_1/2)}{2}\left(\frac{B}{\ell} - A\ell\right) \qquad \text{(A-9a)}$$

$$V_1 \sin \alpha_1 = \frac{\sin(\theta_1/2)}{2}\left(\frac{B}{\ell} + A\ell\right) \qquad \text{(A-9b)}$$

where

$$\ell = (1 + b^2 - 2\ell \sin \gamma)^{1/4} \quad . \qquad \text{(A-9c)}$$

The remaining condition is the cavity closure condition which is equivalent to the requirement of no net sources in the flow. The continuity law requires the volume outflow to be equal to the inflow or with reference to Fig. (6) one has

$$V_1 \cos(\gamma + \alpha_1) = V_2 \cos \gamma \,. \qquad \text{(A-10)}$$

From (A-7) it can be seen that far downstream, $\zeta \longrightarrow \infty$ and $V_2 = C$ there. Thus

$$C = V_1 \cos(\gamma + \alpha)/\cos \gamma$$

and all constants can now be determined in terms of k, α and $c/2\pi$ for a given stagger angle γ . A form convenient for calculation obtained after some manipulation expresses the cavitation number in terms of the parameter b:

$$\sqrt{1+k} = \cos \alpha - \sin \alpha \; \frac{b \cos}{\ell^2 - (1-b)\sin \gamma} \cdot \frac{(1-\ell^2)\cos\gamma - \sqrt{2\ell}\left(1 - \frac{1-b \sin \gamma}{\ell^2}\right)^{1/2} \sin \gamma}{(1+\ell^2)\cos\gamma - \sqrt{2\ell}\left(1 + \frac{1-b \sin\gamma}{\ell^2}\right)^{1/2} \cos\gamma}$$

where ℓ is given by Eq. (A-9).

Discussion. Equations (A-6) and (A-11) constitute the solution of the problem. For given values of b the cavity-spacing ratio is calculated from (A-6) and corresponding values of k are then determined from (A-11). Typical results are shown in Fig. 8 for various values of γ at an angle of attack equal to six degrees.

REFERENCES

1. "On the Pressure Developed in a Liquid During the Collapse of a Spherical Cavity", Rayleigh, Phil. Mag. vol. 34, pg. 94-98, 1917.

2. "Caviation Mechanics and Its Relation to the Design of Hydraulic Equipment", R. T. Knapp, Proc. of the Inst. of Mech. Eng., vol. 166, series A, 1952, pp. 150-163.

3. "The Laws of Cavitation Bubbles at Axially Symmetric Bodies in a Flow", H. Reichardt. Reports and Translations No. 766, Industry of Aircraft Production (1946) distributed by Office of Naval Research, Navy Dept., Washington, D. C.

4. "Jet Wakes and Cavities", G. Birkhoff, E. H. Zarantonello, Academic Press, New York, 1957.

5. "A Theory of Cavitation Flow in Centrifugal-Pump Impellers", C. Gongwer, Trans. ASME, vol. 63, pp. 29-40, 1941.

6. "Phenomenes de la Cavitation", Bulletin de Mai-Juin 1934, Le Laboratoire de Mechanique de L'Ecole Nationale D'Arts et Metiers.

7. "An Experimental Study of Axial Flow Pump Cavitation", P. Guinard, T. Fuller, A. Acosta, Hydrodynamics Laboratory Report E-19.3, Aug. 1953, California Institute of Technology.

8. "An Experimental Study of Cavitating Inducers", A. J. Acosta, second symposium on Naval Hydrodynamics, August 1958, Office of Naval Research, Washington, D. C.

9. "Cavitation in Turbomachines", Wintermitz F. A. L., Parts 1, 2, 3, Water Power, Sept. 1957, pg. 335, October 1957, pg. 378, Nov., pg. 429.

10. "Mechanism of Cavitation Inception and Related Scale-Effects Problem", R. Kermeen, J. McGraw, B. Parkin, Trans. ASME vol. 77, May 1955, pp. 533-541.

11. "Profile Measurements During Cavitation", O. Walchner, Presented before the Congress on Hydromechanical Problems of Ship Propulsion at Hamburg, May 1932. Translated by National Adv. Committee for Aeronautics as Tech. Memo. No. 1060, Washington, D. C.

12. "Fluid Mechanics of Turbo-Machinery", G. F. Wislicenus, McGraw-Hill, New York, 1947, p. 82.

13. "Ten Bladed Kaplan Turbine", Mirko Vitar, Water Power, Dec. 1957, pg. 446-451.

14. "The Design of Axial Flow Pumps", R. D. Bowerman, Trans. ASME, Nov. 1956.

15. "A Study of Pump Rotor Blade Cavitation Parameters", R. C. Brumfield, U. S. Naval Ordnance Test Station, Propulsion Div. Underwater Ord. Dept., Inyokern, China Lake, Calif. T. M. No. 549.

16. "Die Kavitationsgrenze bei Pumpen und Turbinen", C Pfleiderer Zeitschrift Des Vereines Deutschen Ingenieure Band 92, no. 23, pp. 629-635, Aug. 1950.

17. "Principles of Rocket-Turbopump Design", C. C. Ross, American Rocket Soc. Journal, vol. 21, March 1951, pp. 21-33, vol. 78, 1956, pp. 1707-1714.

18. "Development of the Hydraulic Design for the Grand Coulee Pumps", C. Blom, Trans. ASME vol. 72, pp. 53-70, 1950.

19. "Centrifugal and Axial Flow Pumps", A. J. Stepanoff, Wiley, 1958.

20. "Critical Considerations on Cavitation Limits of Centrifugal and Axial Flow Pumps", G. Wislicenus, Trans. ASME, Vol 78, pp. 1707-14.

21. "Berechnung der reibungslosen inkompressiblen Stromung fur vorgegebenes ebenes Schaufelgitter", Schlicting, H. VDI Forschungsheft 447, 1955.

22. "Some Simplified Methods in Airfoil Theory", M. Biot, Jour. Aero. Sci. vol. 9, no. 5, pg. 185, March 1942.

23. "Linearized Theory of Cavity Flow in Two-Dimensions", B. R. Parkin, report P-1745, the Rand Corporation, Santa Monica, California, USA.

24. "Finite Cavity Cascade Flow", Hirsch Cohen, C. O. Sutherland, Math. Report No. 14, April 1958, Dept. of Mathematics, Rensselaer Polytechnic Institute, Troy, New York.

25. "Cavitating Flow Past a Cascade of Circular Air Hydrofoils", A. J. Acosta, Div. of Eng. report E-79.2, California Institute of Technology, in preparation.

26. "Suction Head Correction for Centrifugal Pumps", J. M. Soth, A. Brkich, H. Stahl, Presented to 24th mid-year meeting of American Petroleum Institute's Division of Refining, May, 1959.

27. "Thermodynamic Aspects of Cavitation in Centrifugal Pumps", H. Stahl, A. J. Stepanoff, Trans. ASME vol. 1956, pp. 1691-1693.

28. "Pumping Cryogenic Liquids", R. Jacobs, K. Martin, G. J. Van Wylen, B. W. Birmingham, U. S. Dept. of Commerce, National Bureau of Standards, Boulder Laboratories, Boulder, Colorado, report no. 3569.

29. Discussion by R. C. Fisher, pp. 305-306 of "A Survey of Modern Centrifugal Pump Practice for Oilfield and Oil Refining Services" by N. Tetlow, Proc. of Institution of Mech. Eng., vol. 152, 1945.

30. "Cavitation and NPSH Requirements of Various Liquids", V. Salemann, Trans. ASME series D, June 1959, pp. 167-1731.

31. "On the Dynamics of Small Vapor Bubbles in Liquids", S. Zwick, M. Plesset, J. Math Physics 33, 4, 308-309, Jan. 1955.

32. "Cavitation and Nuclei", R. T. Knapp, Trans. ASME vol. 80, August 1958, pp. 1315-1324.

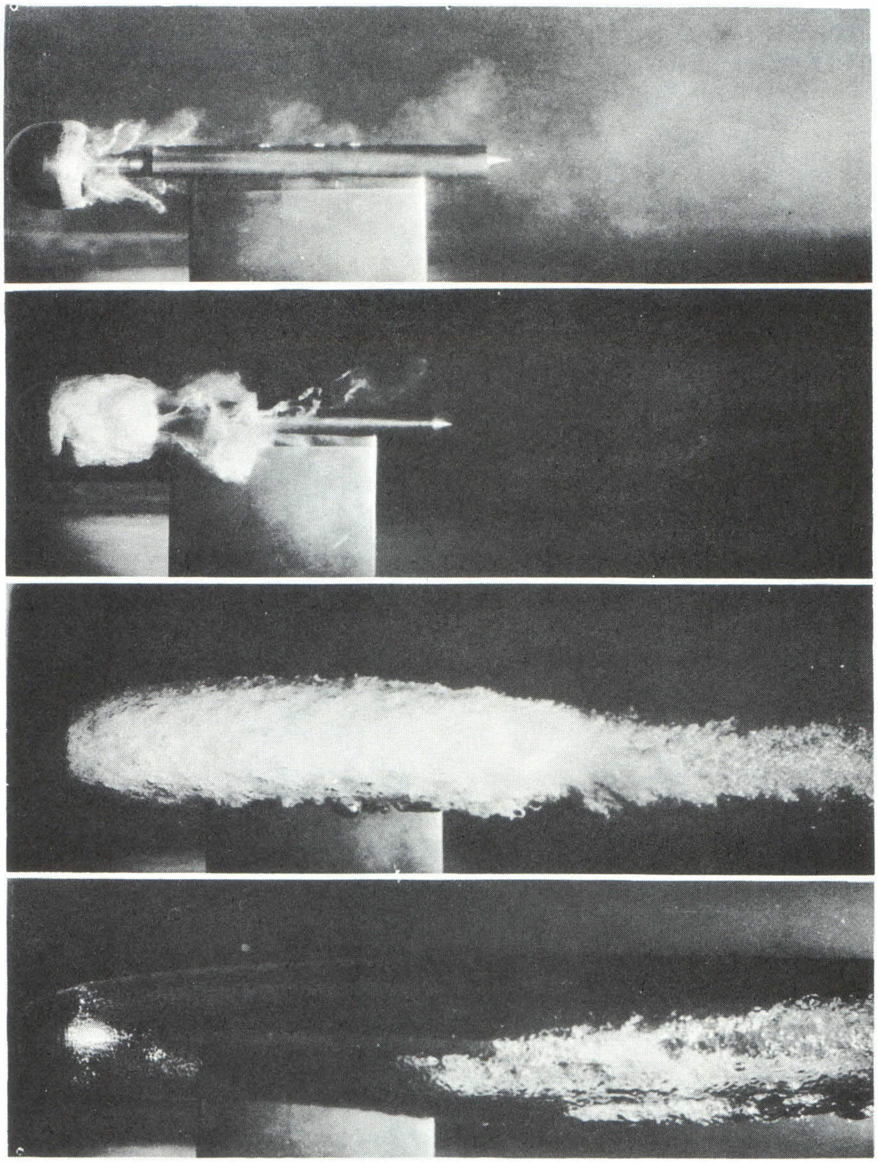


Fig. 1. Progressive Development of Cavitation on a Sphere

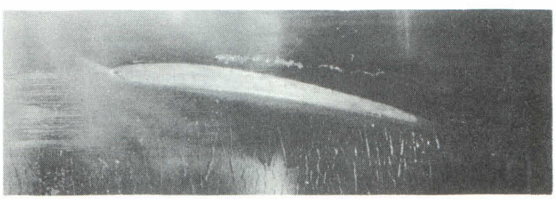
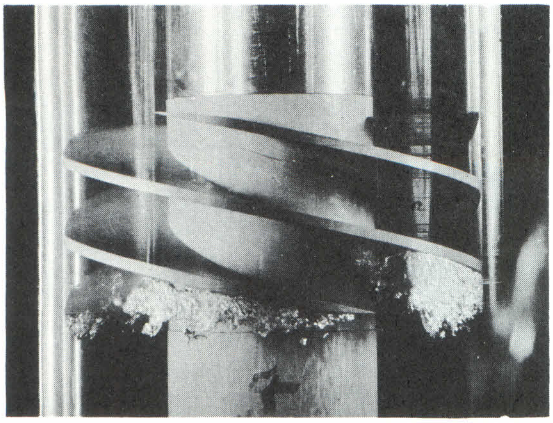
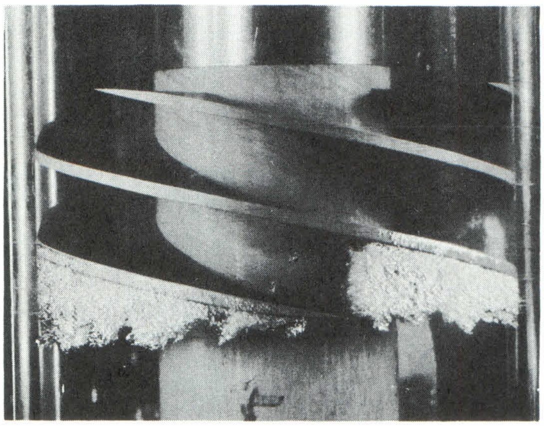


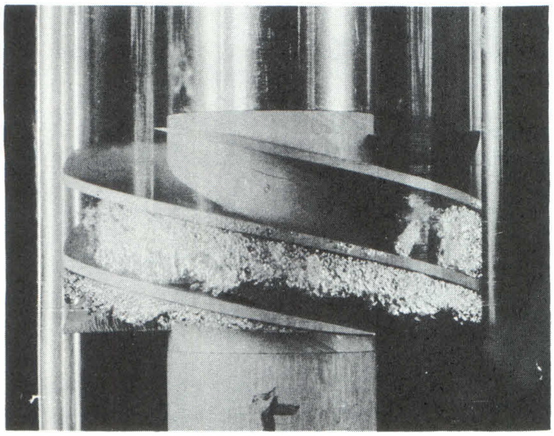
Fig. 2. Tip Vortex Cavitation on an Axial Flow Pump



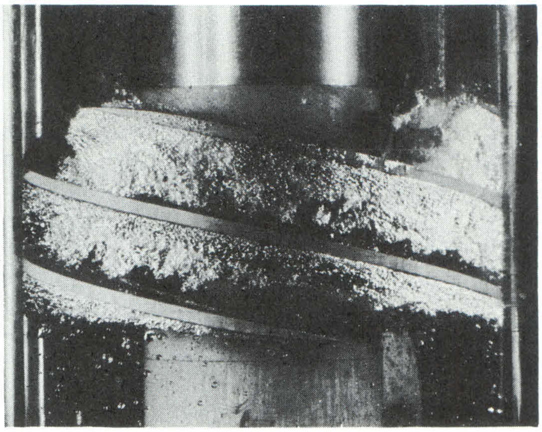
$k = 0.09$



$k = 0.04$



$k = 0.023$



$k = 0.02$

Fig. 3. Development of Cavitation in a 12° Helical Inducer for a Flow Rate Coefficient $\varphi = 0.12$. (These Photos are not Strictly in a Sequence Since They Are Taken at Different Times and Rotative Speeds).

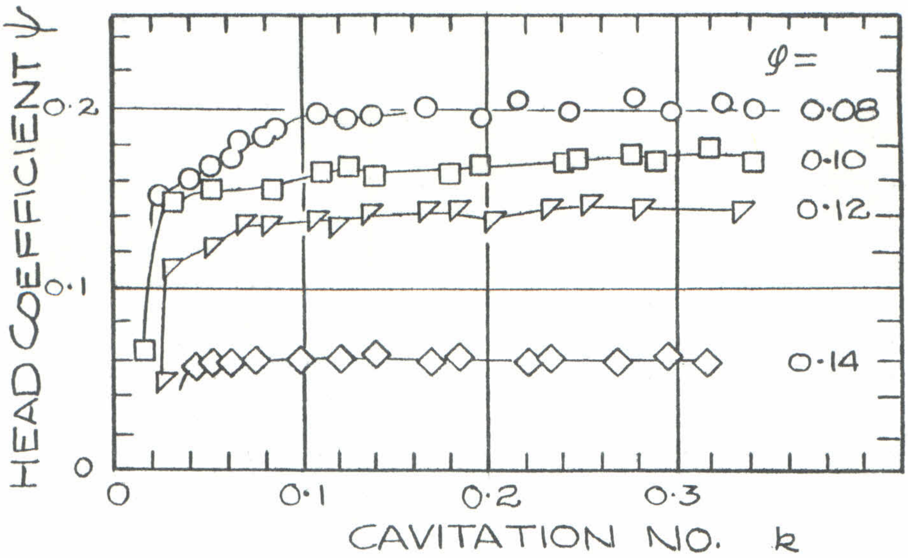


Fig. 4. Head Coefficient vs Cavitation Number at Various Flow Coefficients for an Inducer Type Impeller. The tip blade angle is 12°, the solidity is 2.5 and the hub to case ratio is 0.5.

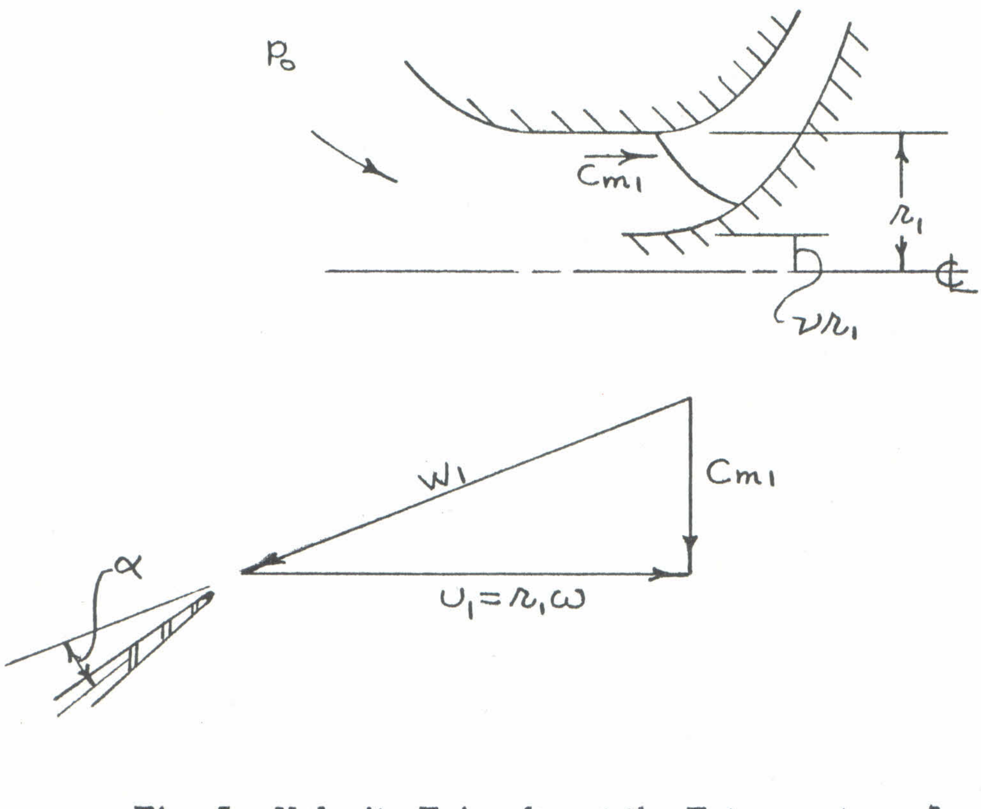


Fig. 5. Velocity Triangles at the Entrance to an Impeller.

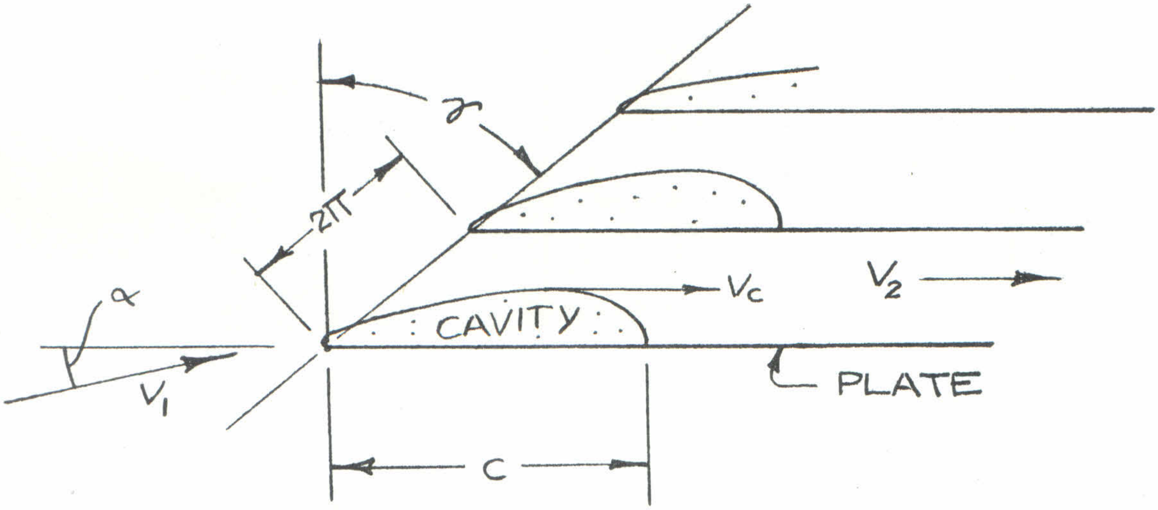


Fig. 6. Sketch showing Partial Cavitation in a Cascade of Semi-Infinite Flat Plates.

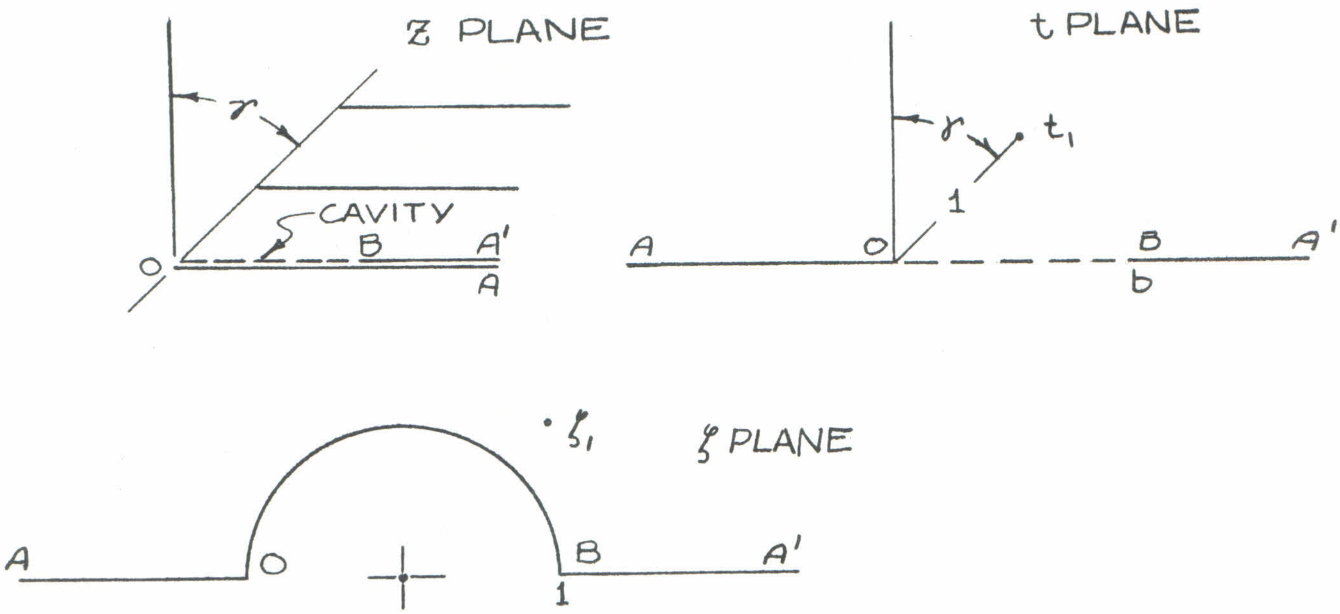


Fig. 7. Sketch Showing Various Transformation Planes.

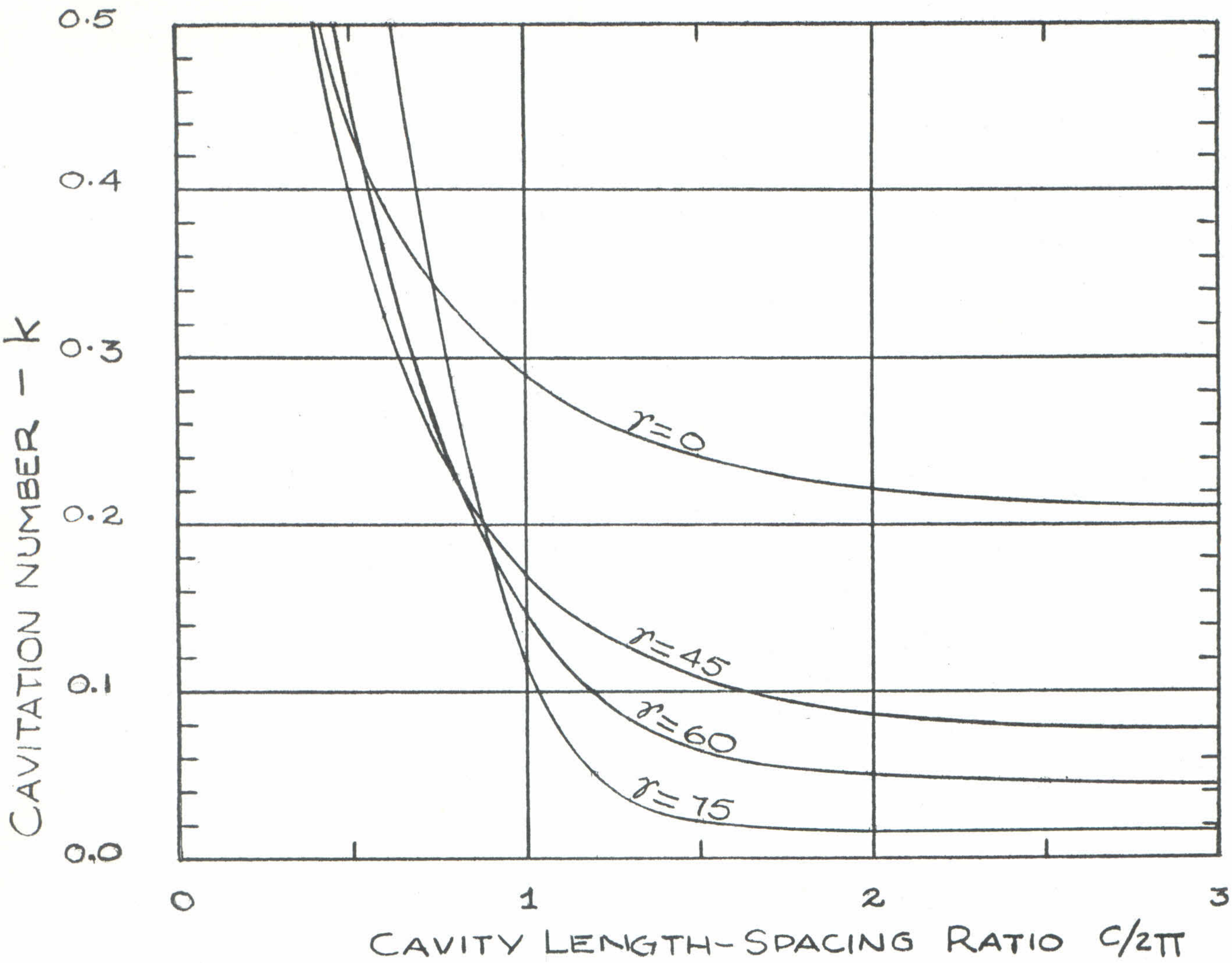


Fig. 8. Cavitation Number vs. Cavity Length to Spacing Ratio for an Angle of Attack of 6 degrees with Various Values of Stagger Angle γ.

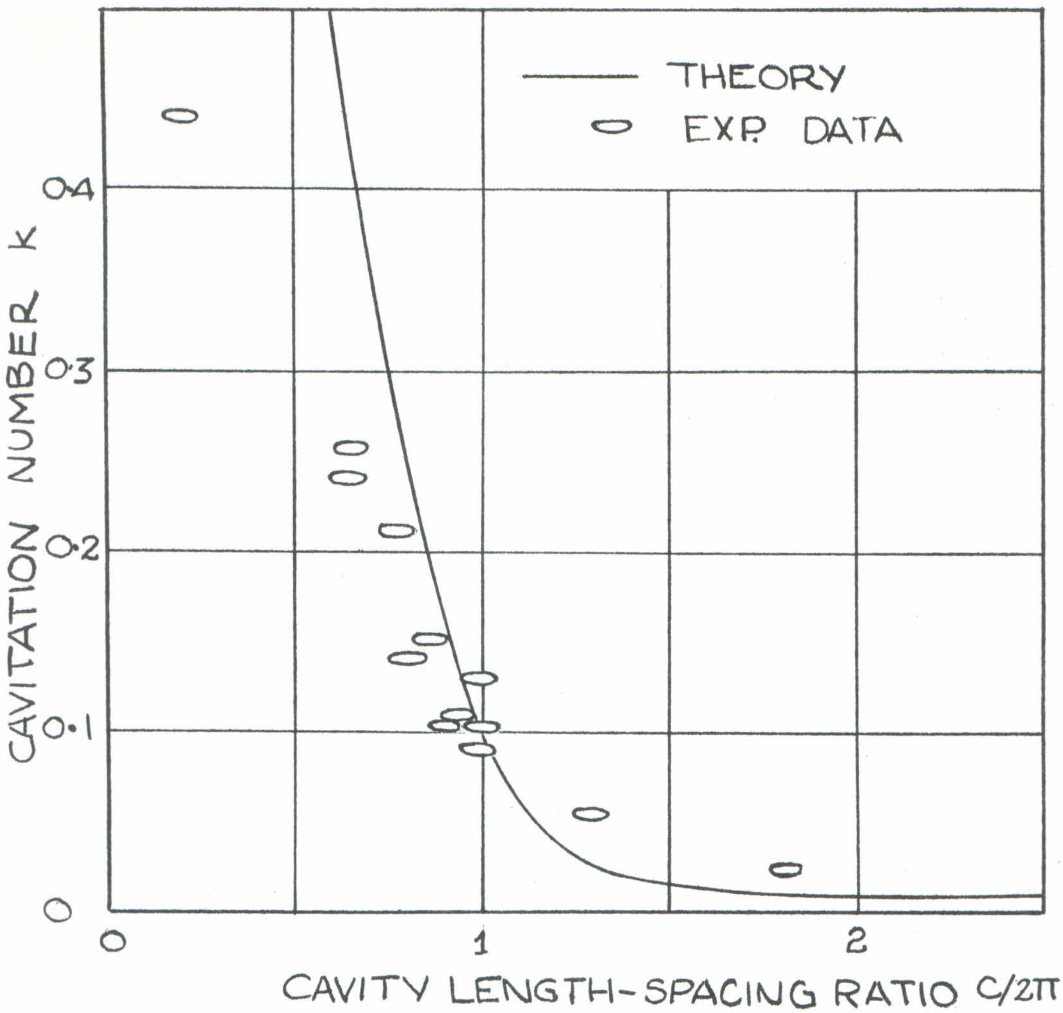


Fig. 9. Cavitation Number vs Cavity Length Compared with Experimental Observations on a Helical Pump Inducer Impeller.

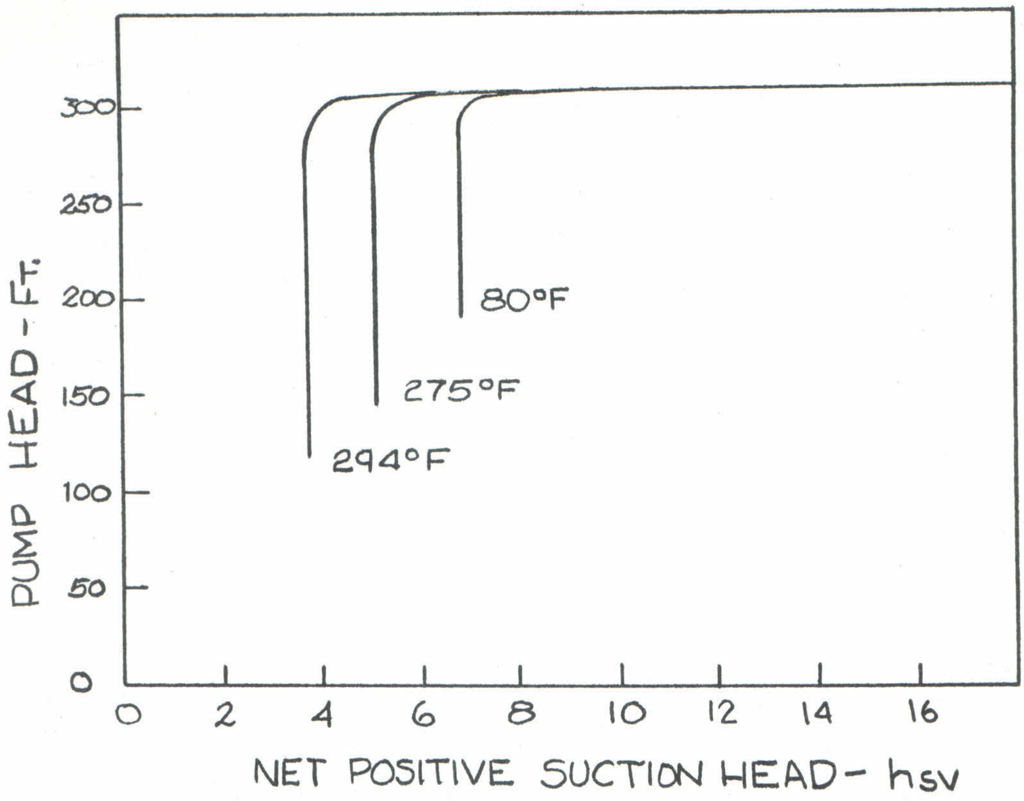


Fig. 10. Pump Head vs Net Positive Suction Head at Constant Flow Rate and Speed for Water at Various Temperatures (after Stahl [10]).

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